Analytical Solutions

Chapter 1

1.1) A lowpass filter is desired with the cutoff frequency of 10 Hz. This filter should attenuate a 100-Hz signal by a factor of at least 78. What should be the order of this filter?

Ignoring the initial slope:

Atten dB = -20 *(N- poles)* log (f_2/f_1) = 20 log 85

-20(*N- poles*) log (10/100) = 20(*N- poles*) log (100/10) = 38.5

N – pole = $\frac{38.5}{2\log 10} = 1.9 \rightarrow 2\ poles$

Chapter 2

2.3) Use Eq. 2.14 to <u>analytically</u> determine the RMS value of a "square wave" with an amplitude of 1.0 volt and a period 0.2 sec.

Apply Eq. 2.14 directly.

$$x(t) = \left[\frac{1}{T}\int_0^T x(t)^2 dt\right]^{1/2} = \left[\frac{1}{0.2}\int_0^{0.2} 1dt\right]^{1/2}$$

$$= \left[5t|_0^{0.2}\right]^{1/2} = [5(0.2)]^{1/2} = 1$$

2.5) Use Eq. 2.14 to <u>analytically</u> determine the RMS value of the waveform shown below with amplitude of 1.0 volt and a period 0.5 sec. [Hint: You can use the symmetry of the waveform to reduce the calculation required.]

To find the RMS of a "sawtooth" wave, apply Eq. 2.14 directly. By symmetry only the first half period need to be evaluated:

$$V_{RMS} = \left[\frac{1}{T}\int_0^T x(t)^2 dt\right]^{1/2} = \left[\frac{1}{0.25}\int_0^{00.25}\left(\frac{3}{0.25}t\right)^2 dt\right]^{1/2}$$

$$V_{RMS} = \left[4\left(\frac{3}{0.25}\right)^2 \frac{t^3}{3}\Big|_0^{0.2}\right]^{1/2} = \left[\frac{4(3)^2(0.35)^3}{(0.25)^2\ 3}\right]^{1/2} = 1.72$$

2.9) If a signal is measured as 2.5 volts and the noise is 28 mV (28 x 10^{-3} volts), what is the SNR in dB.

SNR_{dB} = $20\log\left(\frac{2.5}{0.028}\right)$ = 39 dB

2.10) A single sinusoidal signal is found in a large amount of noise. (If the noise is larger than the signal, the signal is sometimes said to be "buried in noise.") If the RMS value of the noise is 0.5 volts and the SNR is 10 dB, what is the RMS amplitude of the sinusoid?

Apply Eq. 1.13, but solve for the RMS value of the signal:

SNR_{dB} = $20\log\left(\frac{V_{sig}}{Vnoise}\right)$ 10 dB; $20log\left(\frac{V_{sig}}{0.5}\right) 10$

$\log\left(\frac{V_{sig}}{0.5}\right) = \frac{10}{20} = 0.5 \qquad \frac{V_{sig}}{0.5} = 10^{0.5} = 3.16 \qquad V_{sig}$ = 1.58 volts rms

2.12) An 8-bit ADC converter that has an input range of ± 5 volts is used to convert a signal that ranges between ± 2 volts. What is the SNR of the input if the input noise equals the quantization noise of the converter? [Hint: Refer back to Eq. 1.8 to find the quantization noise.]

$$\sigma^2 = \frac{V_{max}^2}{12(2^N - 1)} = \frac{10^2}{12(2^8 - 1)} = \frac{100}{3060} = 0.0327$$

$$V_{noise} \approx \sigma = \sqrt{0.0327} = 0.18\ volts$$

$$SNR = \frac{2}{0.18} = 11.11 = 20.9\ dB$$

2.16) A resistor produces 10 µV noise (i.e., 10 x 10^{-6} volts noise) when the room temperature is 310K and the bandwidth is 1 kHz (i.e., 1000 Hz). What current noise would be produced by this resistor?

Need to find *R* from information on the voltage noise of the resistor. Using Eq. 2.25 for the voltage noise from a resistor:

$V_n = \sqrt{4ktRBW}$ = 10 x 10^{-6} volts

$R = \frac{(10^{-5})^2}{4kT\ BW} = \frac{10^{-10}}{1.7\times10^{-20}(10^3)}$ = 5.88 x 10^6 = 5.88 MΩ

i_n = $\sqrt{4ktBW/R}$ = $\sqrt{1.7\times10^{-20}(10^3)/5.88\times10^6} = 1.7\times10^{-12}$ amp

2.17) The noise voltage out of a 1-MΩ (i.e., 10^6-Ω) resistor is measured using a digital voltmeter as 1.5 µV at a room temperature of 310 K. What is the effective bandwidth of the voltmeter?

Apply the equation for noise from a resistor (Eq. 2.25), but solve for bandwidth:

$V_n = \sqrt{4ktRBW}$ $\quad BW = \frac{v_n^2}{4kTR} = \frac{(1.5\times10^{-6})^2}{1.7\times10^{-20}(10^6)}$ = 132 Hz

2.18) A 3-ma current flows through both a diode (i.e., a semiconductor) and a 20,000-Ω (i.e. 20-kΩ) resistor. What is the net current noise, i_n? Assume a bandwidth of 1 kHz (i.e., 1 x10^3 Hz). Which of the two components is responsible for producing the most noise?

There are two current noise sources. Apply the appropriate current noise equations, Eq. 2.26 and 2.27, and combine as the square root of the sum of squares (modified Eq. 2.28):

$$i_n = \left(i_j^2 + i_D^2\right)^{1/2} = \left(\frac{4kTBW}{R} + 2qI_DBW\right)^{1/2}$$

$$= \left(1.76 \times 10^{-20}\left(\frac{10^3}{20 \times 10^3}\right) + 2(1.662 \times 10^{-19})3 \times 10^3(10^3)\right)^{1/2}$$

$i_n = (8.5 \times 10^{-22} + 9.96 \times 10^{-19})^{1/2}$ = 9.98 x 10^{-10} amps

---

Chapter 10

10.13) From inspection of the Hénon map equations, determine the best values for embedding dimension and delay.

It is apparent from the structure of the Hénon map equations that an ideal embedding dimension and delay should be 2 dimensions and one sample.

10.17) How would the presence of broadband noise in a signal of interest affect your estimation of the optimal delay? Would you expect the same behavior if the signal is linear compared to nonlinear? What about a comparison of stochastic versus deterministic systems?

Broadband noise decreases the length of time that a series is correlated. If the SNR is too low, the correlation will fall to too few samples to perform a meaningful or accurate phase-space reconstruction. This will happen regardless of whether or not a system is linear or nonlinear, but a nonlinear system can also have its nonlinear properties masked by noise, making it appear more similar to a linear system. Similarly, noise will make it difficult to determine an optimal delay for both a stochastic and dynamical system.