## 13-1.

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_{1}=\{2 \mathbf{i}+6 \mathbf{j}-2 t \mathbf{k}\} \mathrm{lb}, \quad \mathbf{F}_{2}=$ $\left\{t^{2} \mathbf{i}-4 t \mathbf{j}-1 \mathbf{k}\right\} \mathrm{lb}$, and $\mathbf{F}_{3}=\{-2 t \mathbf{i}\} \mathrm{lb}$, where $t$ is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.


## SOLUTION

$\Sigma \mathbf{F}=m \mathbf{a} ; \quad(2 \mathbf{i}+6 \mathbf{j}-2 t \mathbf{k})+\left(t^{2} \mathbf{i}-4 t \mathbf{j}-1 \mathbf{k}\right)-2 t \mathbf{i}-6 \mathbf{k}=\left(\frac{6}{32.2}\right)\left(\mathbf{a}_{x} \mathbf{i}+a_{y} \mathbf{j}+a_{z} \mathbf{k}\right)$
Equating components:
$\left(\frac{6}{32.2}\right) a_{x}=t^{2}-2 t+2\left(\frac{6}{32.2}\right) a_{y}=-4 t+6\left(\frac{6}{32.2}\right) a_{z}=-2 t-7$
Since $d v=a d t$, integrating from $\nu=0, t=0$, yields
$\left(\frac{6}{32.2}\right) v_{x}=\frac{t^{3}}{3}-t^{2}+2 t \quad\left(\frac{6}{32.2}\right) v_{y}=-2 t^{2}+6 t \quad\left(\frac{6}{32.2}\right) v_{z}=-t^{2}-7 t$
Since $d s=v d t$, integrating from $s=0, t=0$ yields
$\left(\frac{6}{32.2}\right) s_{x}=\frac{t^{4}}{12}-\frac{t^{3}}{3}+t^{2}\left(\frac{6}{32.2}\right) s_{y}=-\frac{2 t^{3}}{3}+3 t^{2} \quad\left(\frac{6}{32.2}\right) s_{z}=-\frac{t^{3}}{3}-\frac{7 t^{2}}{2}$
When $t=2 \mathrm{~s}$ then $, \quad s_{x}=14.31 \mathrm{ft}, \quad s_{y}=35.78 \mathrm{ft} \quad s_{z}=-89.44 \mathrm{ft}$
Thus,

$s=\sqrt{(14.31)^{2}+(35.78)^{2}+(-89.44)^{2}}=97.4 \mathrm{ft}$
Ans.

## Ans:

$s=97.4 \mathrm{ft}$

## 13-2.

The two boxcars $A$ and $B$ have a weight of 20000 lb and 30000 lb , respectively. If they are freely coasting down the incline when the brakes are applied to all the wheels of car $A$, determine the force in the coupling $C$ between the two cars. The coefficient of kinetic friction between the wheels of $A$ and the tracks is $\mu_{k}=0.5$. The wheels of car $B$ are free to roll. Neglect their mass in the calculation. Suggestion: Solve the problem by representing single resultant normal forces acting on $A$ and $B$, respectively.

## SOLUTION

## $\operatorname{Car} A$ :

$$
\begin{array}{ll}
+\nwarrow \Sigma F_{y}=0 ; & N_{A}-20000 \cos 5^{\circ}=0 \quad N_{A}=19923.89 \mathrm{lb} \\
+\nearrow \Sigma F_{x}=m a_{x} ; & 0.5(19923.89)-T-20000 \sin 5^{\circ}=\left(\frac{20000}{32.2}\right) a
\end{array}
$$

Both cars:
$+\nearrow \Sigma F_{x}=m a_{x} ; \quad 0.5(19923.89)-50000 \sin 5^{\circ}=\left(\frac{50000}{32.2}\right) a$
Solving,

$$
\begin{aligned}
& a=3.61 \mathrm{ft} / \mathrm{s}^{2} \\
& T=5.98 \mathrm{kip}
\end{aligned}
$$


(1)


Ans.

Ans:
$T=5.98 \mathrm{kip}$

## 13-3.

If the coefficient of kinetic friction between the $50-\mathrm{kg}$ crate and the ground is $\mu_{k}=0.3$, determine the distance the crate travels and its velocity when $t=3 \mathrm{~s}$. The crate starts from rest, and $P=200 \mathrm{~N}$.


## SOLUTION

Free-Body Diagram: The kinetic friction $F_{f}=\mu_{k} N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. $a$.

Equations of Motion: Here, $a_{y}=0$. Thus,

$$
\begin{array}{cc}
+\uparrow \Sigma F_{y}=0 ; & N-50(9.81)+200 \sin 30^{\circ}=0 \\
& N=390.5 \mathrm{~N} \\
\pm \Sigma F_{x}=m a_{x} ; & 200 \cos 30^{\circ}-0.3(390.5)=50 a \\
a=1.121 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

Kinematics: Since the acceleration a of the crate is constant,

(a)
$(\xrightarrow{\rightarrow}) \quad v=v_{0}+a_{c} t$

$$
v=0+1.121(3)=3.36 \mathrm{~m} / \mathrm{s}
$$

Ans.
and

$$
\begin{aligned}
(\xrightarrow{\rightarrow}) \quad s & =s_{0}+v_{0} t+\frac{1}{2} a_{c} t^{2} \\
s & =0+0+\frac{1}{2}(1.121)\left(3^{2}\right)=5.04 \mathrm{~m}
\end{aligned}
$$

Ans.

> Ans:
> $v=3.36 \mathrm{~m} / \mathrm{s}$
> $s=5.04 \mathrm{~m}$
*13-4.
If the $50-\mathrm{kg}$ crate starts from rest and achieves a velocity of $v=4 \mathrm{~m} / \mathrm{s}$ when it travels a distance of 5 m to the right, determine the magnitude of force $\mathbf{P}$ acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_{k}=0.3$.


## SOLUTION

Kinematics: The acceleration a of the crate will be determined first since its motion is known.

$$
\begin{aligned}
& \left(\begin{array}{l} 
\pm \\
)
\end{array} \quad v^{2}=v_{0}^{2}+2 a_{c}\left(s-s_{0}\right)\right. \\
& 4^{2}=0^{2}+2 a(5-0) \\
& a
\end{aligned}
$$

Free-Body Diagram: Here, the kinetic friction $F_{f}=\mu_{k} N=0.3 N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. $a$.

## Equations of Motion:

$$
\begin{aligned}
+\uparrow \Sigma F_{y}=m a_{y} ; & N+P \sin 30^{\circ}-50(9.81)=50(0) \\
& N=490.5-0.5 P
\end{aligned}
$$



Using the results of $\mathbf{N}$ and $\mathbf{a}$,
$\xrightarrow{\text { د }} \Sigma F_{x}=m a_{x} ; \quad P \cos 30^{\circ}-0.3(490.5-0.5 P)=50(1.60)$

$$
P=224 \mathrm{~N}
$$

Ans.

Ans:
$P=224 \mathrm{~N}$

