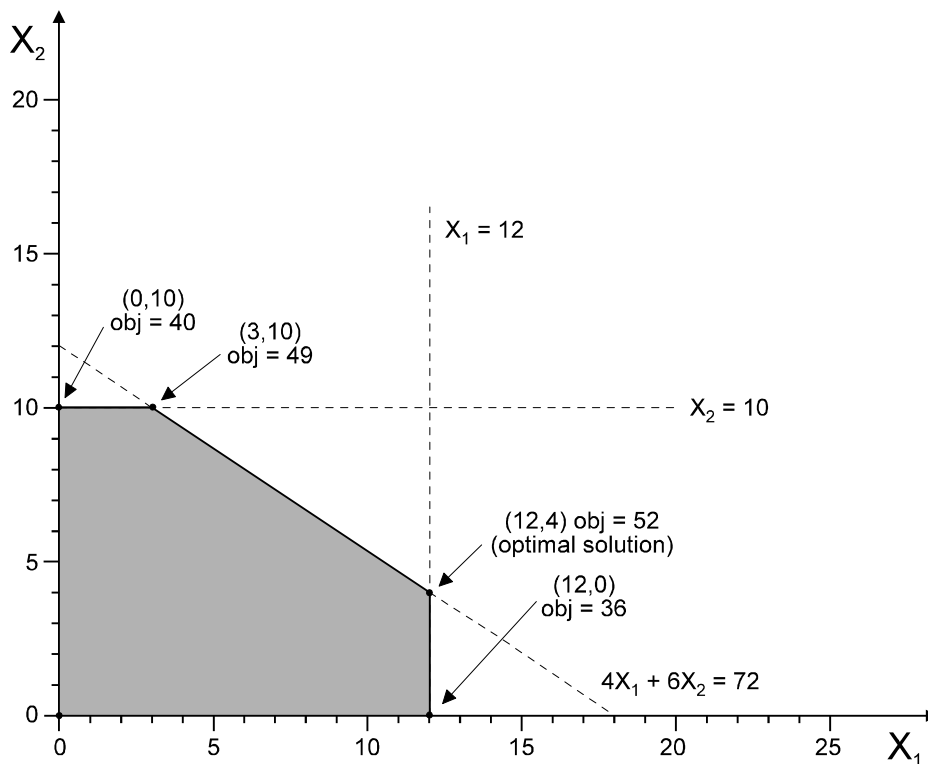


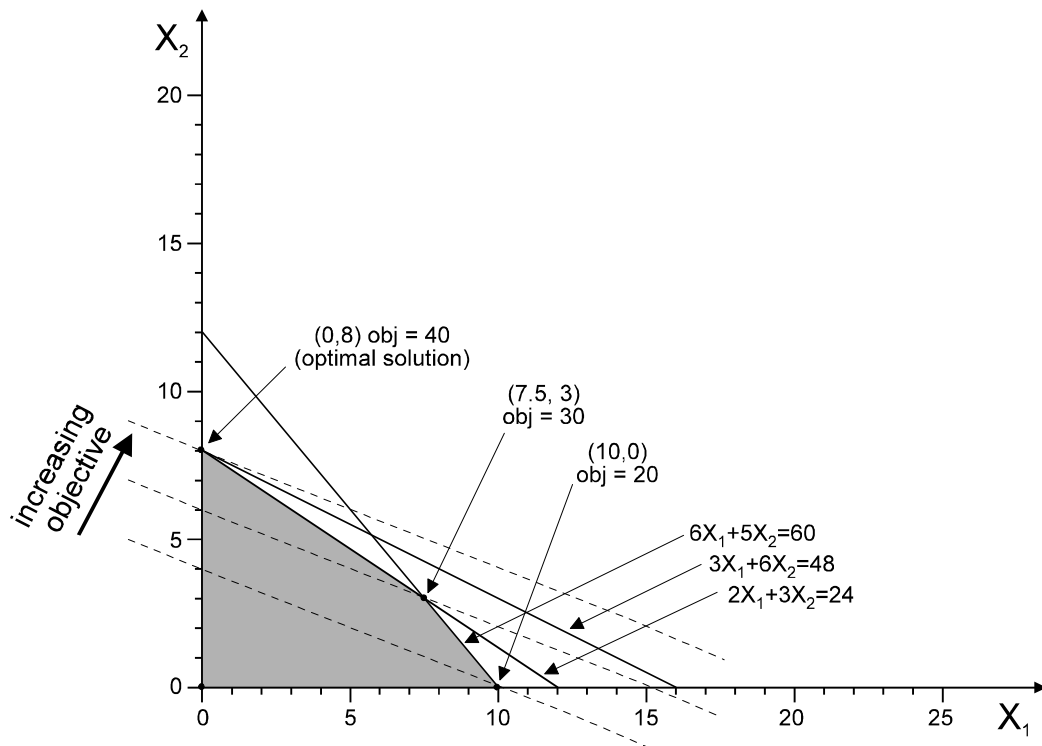
## Chapter 2

### Introduction to Optimization & Linear Programming

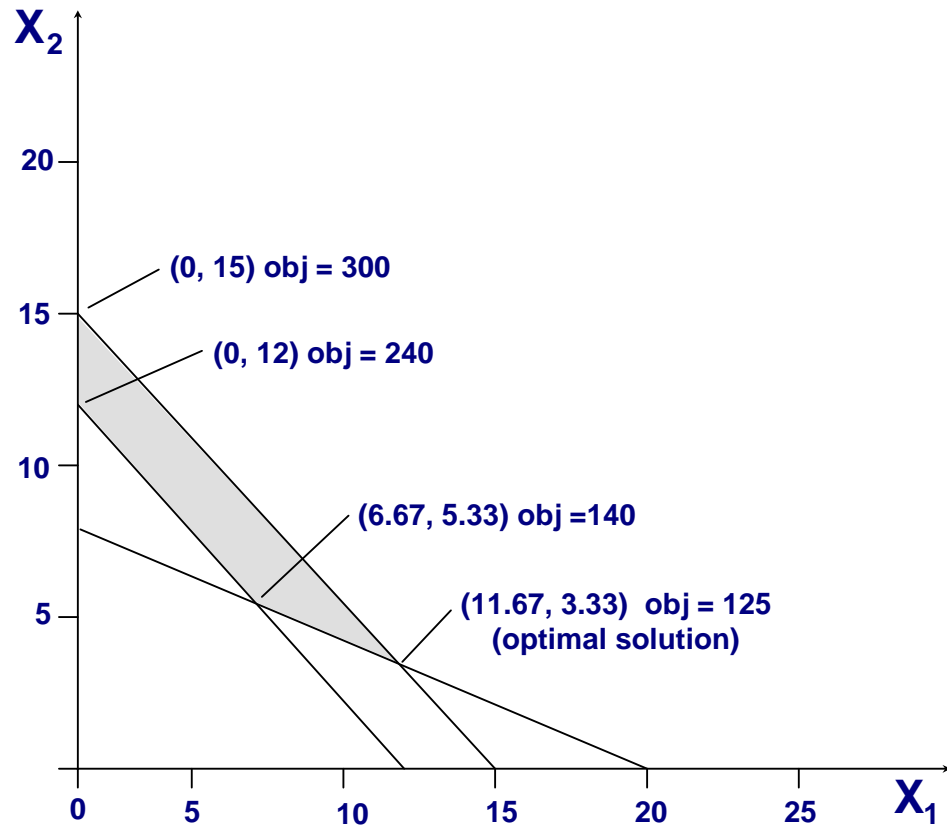
1. If an LP model has more than one optimal solution it has an *infinite* number of alternate optimal solutions. In Figure 2.8, the two extreme points at (122, 78) and (174, 0) are alternate optimal solutions, but there are an infinite number of alternate optimal solutions along the edge connecting these extreme points. This is true of all LP models with alternate optimal solutions.
2. There is no guarantee that the optimal solution to an LP problem will occur at an integer-valued extreme point of the feasible region. (An exception to this general rule is discussed in Chapter 5 on networks).
3. We can graph an inequality as if they were an equality because the condition imposed by the equality corresponds to the boundary line (or most extreme case) of the inequality.
4. The objectives are equivalent. For any values of  $X_1$  and  $X_2$ , the absolute value of the objectives are the same. Thus, maximizing the value of the first objective is equivalent to minimizing the value of the second objective.
5.
  - a. linear
  - b. nonlinear
  - c. linear, can be re-written as:  $4 X_1 - .3333 X_2 = 75$
  - d. linear, can be re-written as:  $2.1 X_1 + 1.1 X_2 - 3.9 X_3 \leq 0$
  - e. nonlinear
- 6.



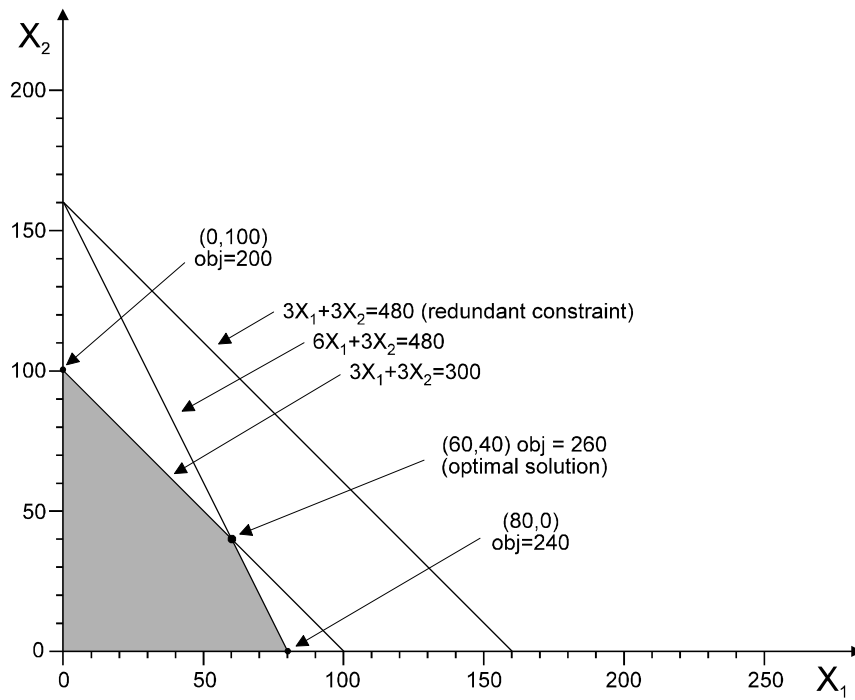
7.



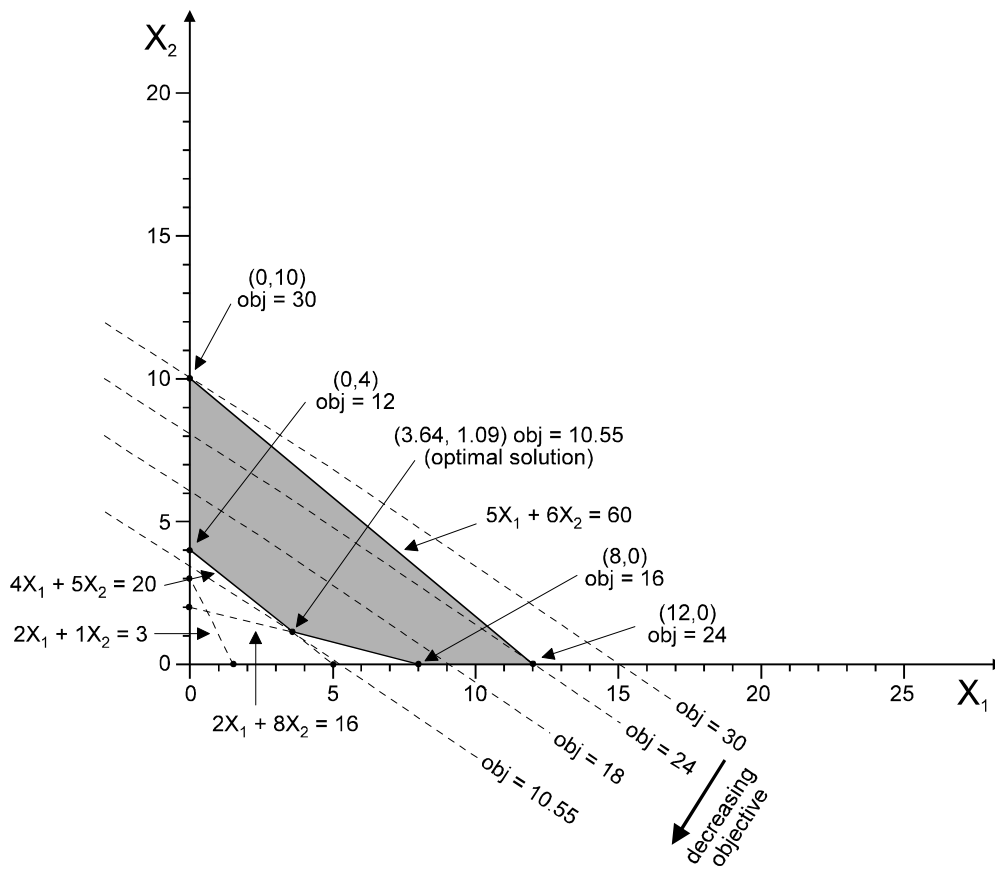
8.



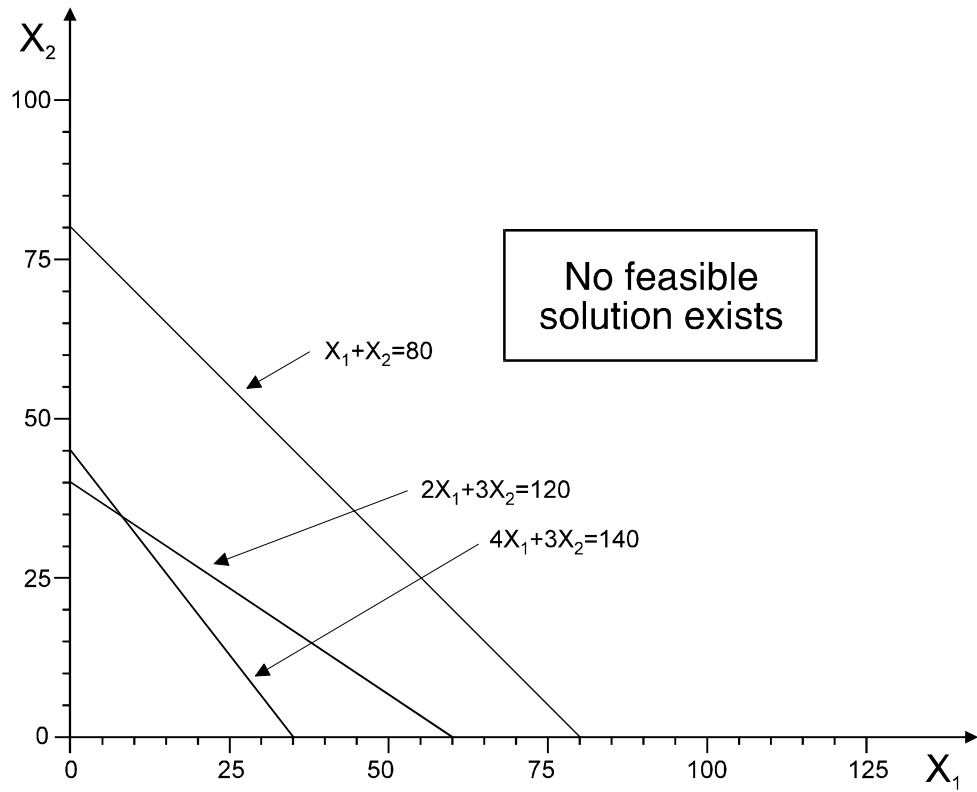
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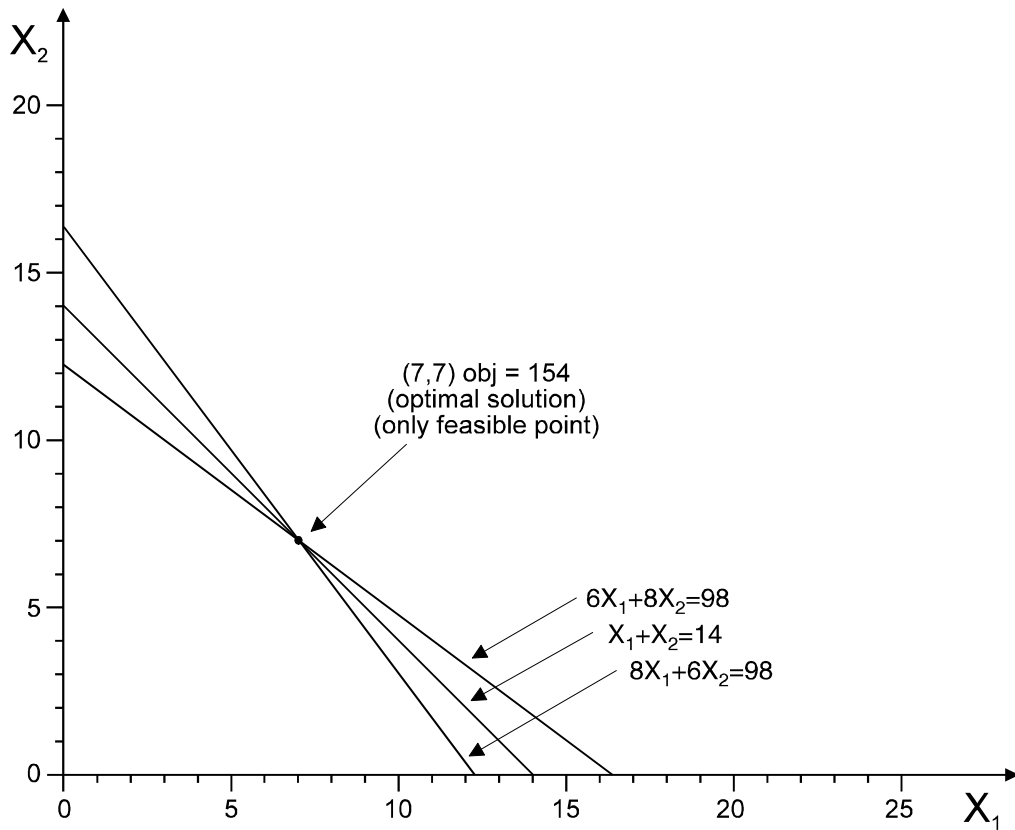
10.



11.

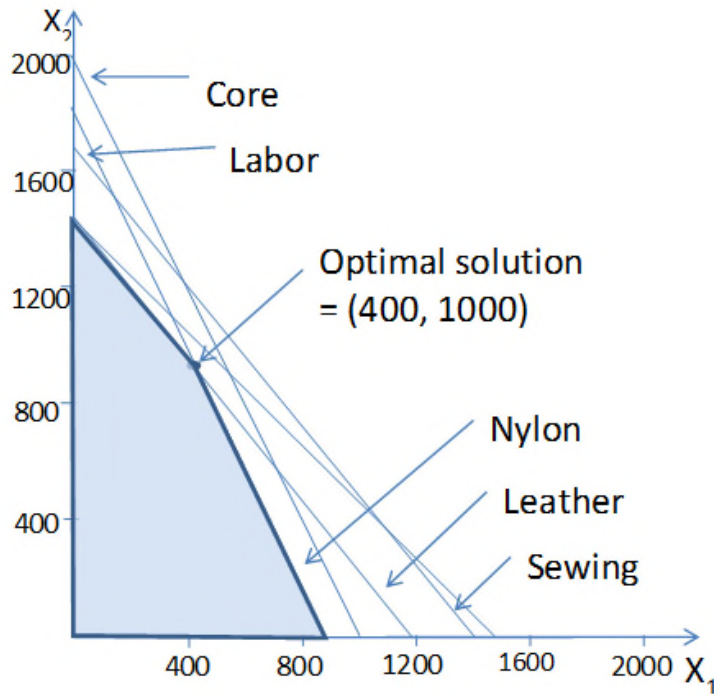


12.



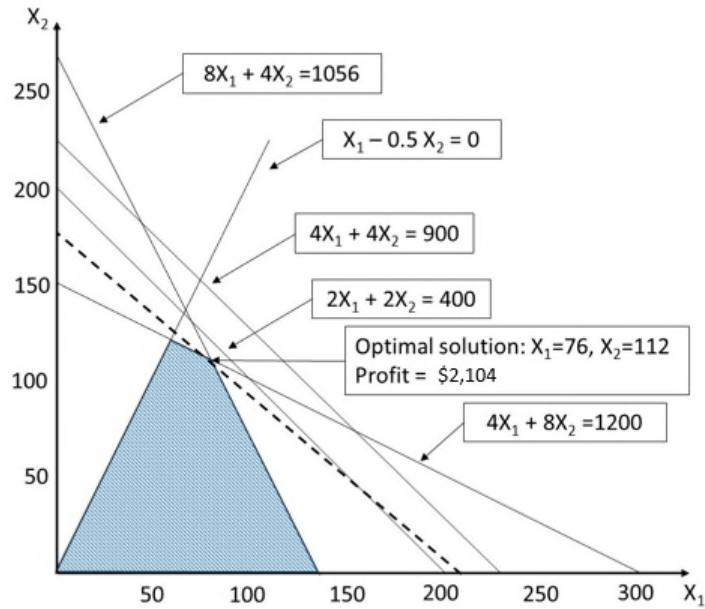
13.  $X_1$  = number of softballs to produce,  $X_2$  = number of baseballs to produce

$$\begin{array}{ll} \text{MAX} & 6 X_1 + 4.5 X_2 \\ \text{ST} & 5 X_1 + 4 X_2 \leq 6000 \\ & 6 X_1 + 3 X_2 \leq 5400 \\ & 4 X_1 + 2 X_2 \leq 4000 \\ & 2.5 X_1 + 2 X_2 \leq 3500 \\ & 1 X_1 + 1 X_2 \leq 1500 \\ & X_1, X_2 \geq 0 \end{array}$$



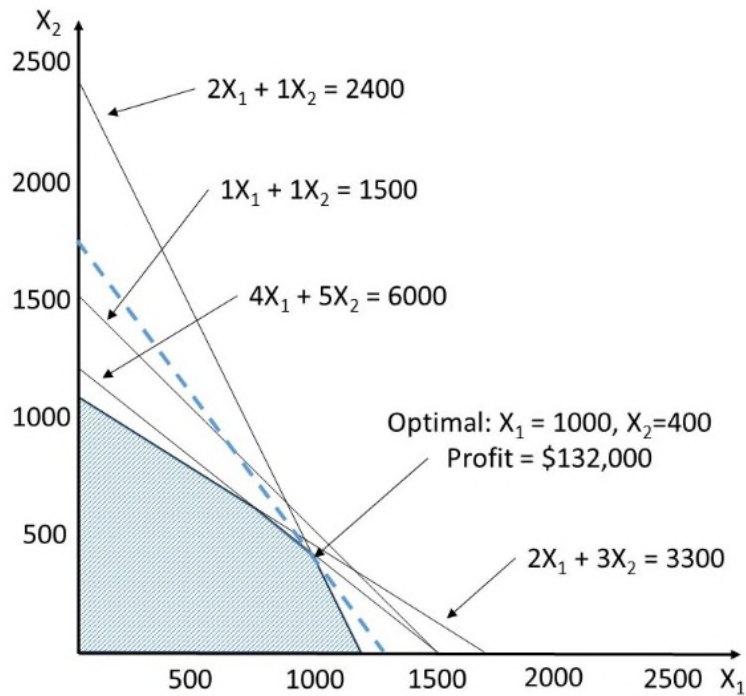
14.  $X_1$  = number of His chairs to produce,  $X_2$  = number of Hers chairs to produce

$$\begin{array}{ll} \text{MAX} & 10 X_1 + 12 X_2 \\ \text{ST} & 4 X_1 + 8 X_2 \leq 1200 \\ & 8 X_1 + 4 X_2 \leq 1056 \\ & 2 X_1 + 2 X_2 \leq 400 \\ & 4 X_1 + 4 X_2 \leq 900 \\ & 1 X_1 - 0.5 X_2 \geq 0 \\ & X_1, X_2 \geq 0 \end{array}$$



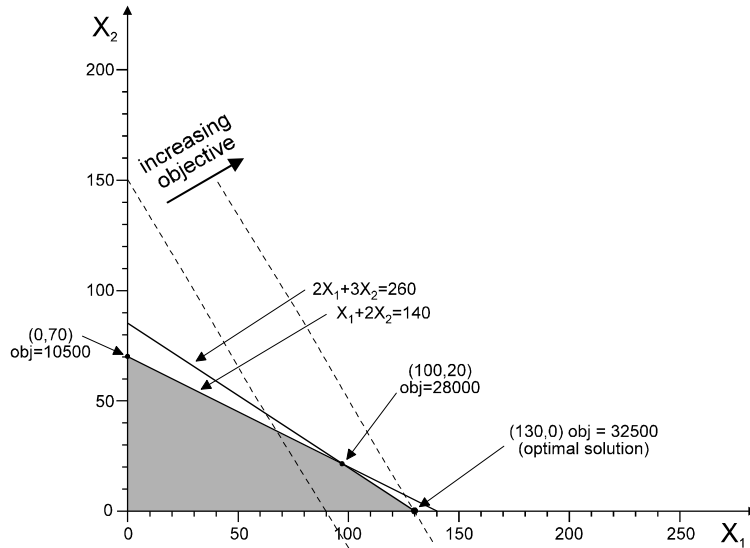
15.  $X_1$  = number of propane grills to produce,  $X_2$  = number of electric grills to produce

$$\begin{aligned} \text{MAX} \quad & 100 X_1 + 80 X_2 \\ \text{ST} \quad & 2 X_1 + 1 X_2 \leq 2400 \\ & 4 X_1 + 5 X_2 \leq 6000 \\ & 2 X_1 + 3 X_2 \leq 3300 \\ & 1 X_1 + 1 X_2 \leq 1500 \\ & X_1, X_2 \geq 0 \end{aligned}$$



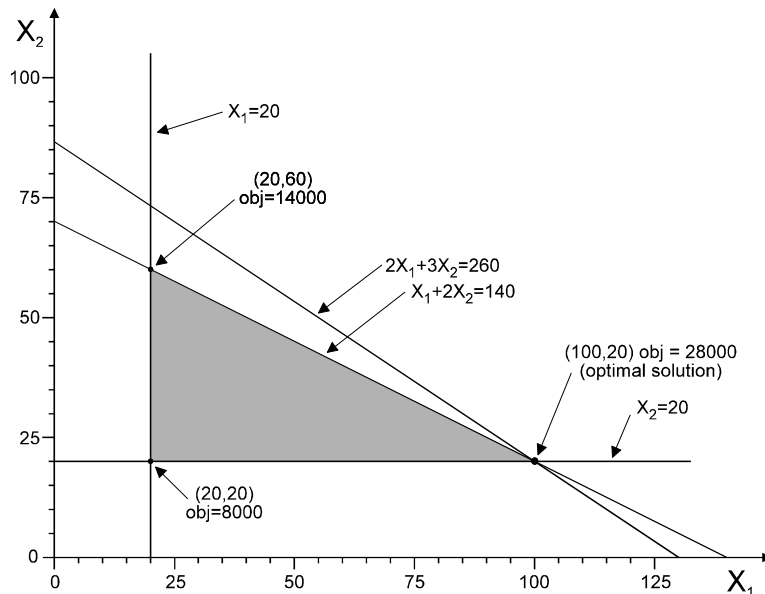
16.  $X_1$  = number of generators,  $X_2$  = number of alternators

$$\begin{array}{ll} \text{MAX} & 250 X_1 + 150 X_2 \\ \text{ST} & 2 X_1 + 3 X_2 \leq 260 \\ & 1 X_1 + 2 X_2 \leq 140 \\ & X_1, X_2 \geq 0 \end{array}$$



17.  $X_1$  = number of generators,  $X_2$  = number of alternators

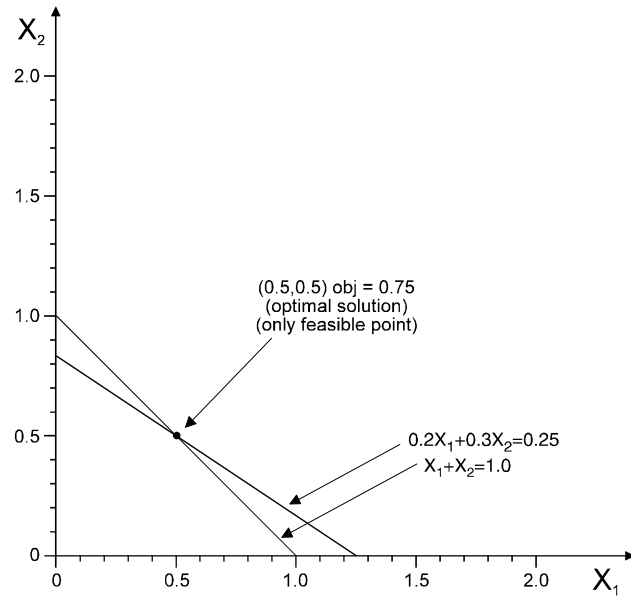
$$\begin{array}{ll} \text{MAX} & 250 X_1 + 150 X_2 \\ \text{ST} & 2 X_1 + 3 X_2 \leq 260 \\ & 1 X_1 + 2 X_2 \leq 140 \\ & X_1 \geq 20 \\ & X_2 \geq 20 \end{array}$$



- d. No, the feasible region would not increase so the solution would not change -- you'd just have extra (unused) wiring capacity.

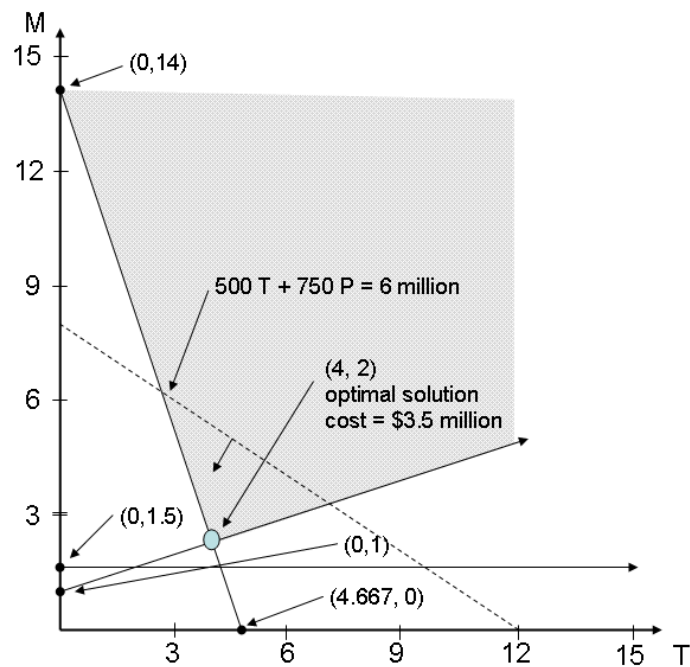
18.  $X_1$  = proportion of beef in the mix,  $X_2$  = proportion of pork in the mix

$$\begin{array}{ll}\text{MIN} & .85 X_1 + .65 X_2 \\ \text{ST} & 1X_1 + 1X_2 = 1 \\ & 0.2 X_1 + 0.3 X_2 \leq 0.25 \\ & X_1, X_2 \geq 0\end{array}$$



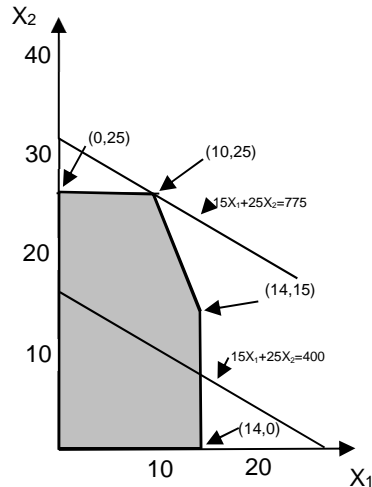
19. T = number of TV ads to run, M = number of magazine ads to run

$$\begin{array}{ll}\text{MIN} & 500 T + 750 P \\ \text{ST} & 3T + 1P \geq 14 \\ & -1T + 4P \geq 4 \\ & 0T + 2P \geq 3 \\ & T, P \geq 0\end{array}$$



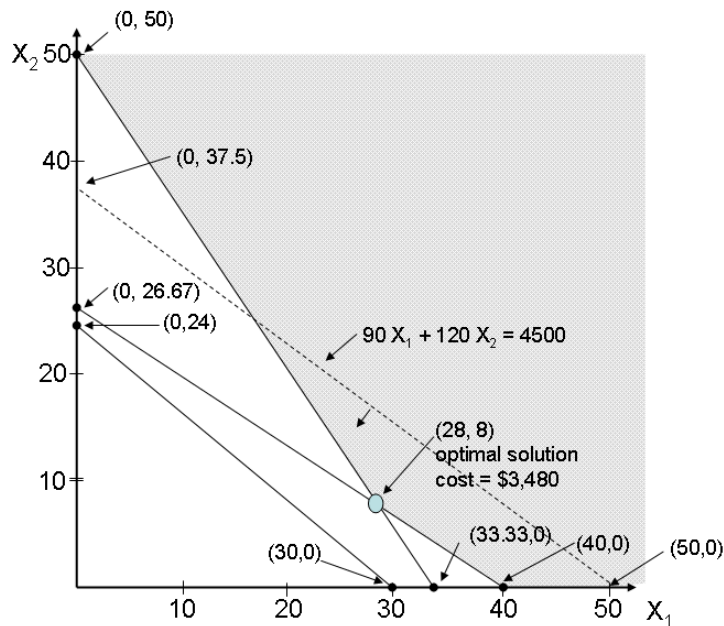
20.  $X_1$  = # of TV spots,  $X_2$  = # of magazine ads

$$\begin{array}{lll} \text{MAX} & 15 X_1 + 25 X_2 & \text{(profit)} \\ \text{ST} & 5 X_1 + 2 X_2 \leq 100 & \text{(ad budget)} \\ & 5 X_1 + 0 X_2 \leq 70 & \text{(TV limit)} \\ & 0 X_1 + 2 X_2 \leq 50 & \text{(magazine limit)} \\ & X_1, X_2 \geq 0 \end{array}$$



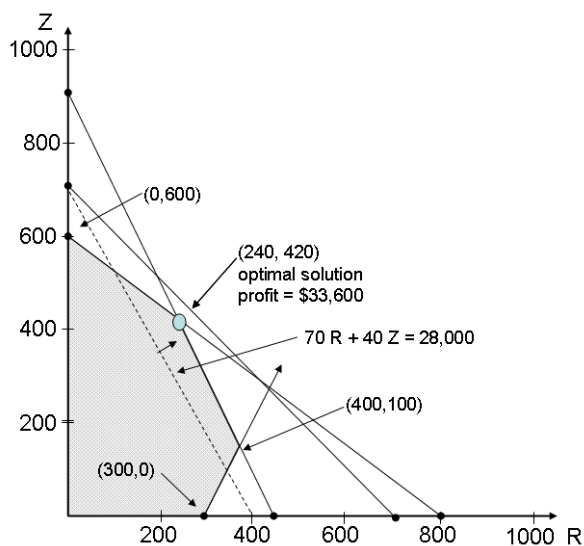
21.  $X_1$  = tons of ore purchased from mine 1,  $X_2$  = tons of ore purchased from mine 2

$$\begin{array}{lll} \text{MIN} & 90 X_1 + 120 X_2 & \text{(cost)} \\ \text{ST} & 0.2 X_1 + 0.3 X_2 \geq 8 & \text{(copper)} \\ & 0.2 X_1 + 0.25 X_2 \geq 6 & \text{(zinc)} \\ & 0.15 X_1 + 0.1 X_2 \geq 5 & \text{(magnesium)} \\ & X_1, X_2 \geq 0 \end{array}$$



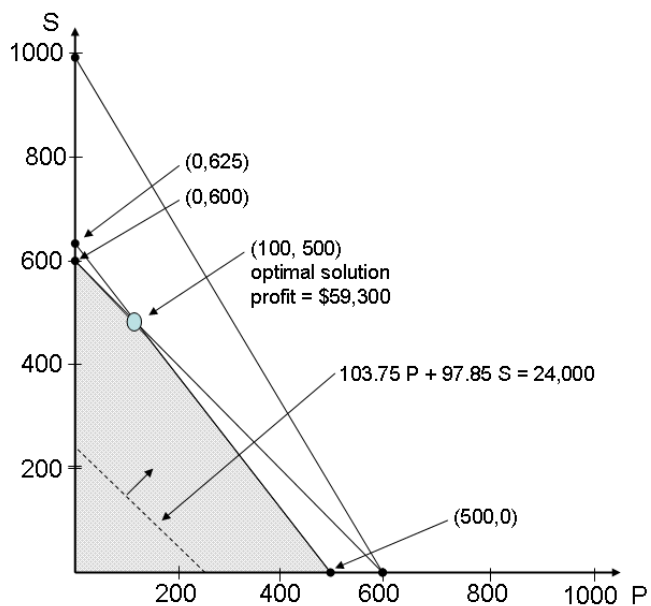
22. R = number of Razors produced, Z = number of Zoomers produced

$$\begin{array}{ll}\text{MAX} & 70R + 40Z \\ \text{ST} & R + Z \leq 700 \\ & R - Z \leq 300 \\ & 2R + 1Z \leq 900 \\ & 3R + 4Z \leq 2400 \\ & R, Z \geq 0\end{array}$$



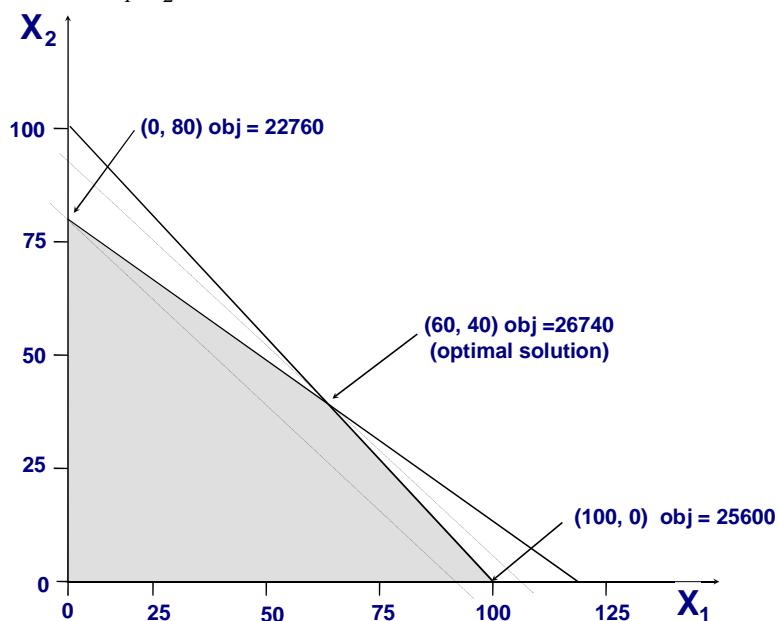
23. P = number of Presidential desks produced, S = number of Senator desks produced

$$\begin{array}{ll}\text{MAX} & 103.75P + 97.85S \\ \text{ST} & 30P + 24S \leq 15,000 \\ & 1P + 1S \leq 600 \\ & 5P + 3S \leq 3000 \\ & P, S \geq 0\end{array}$$



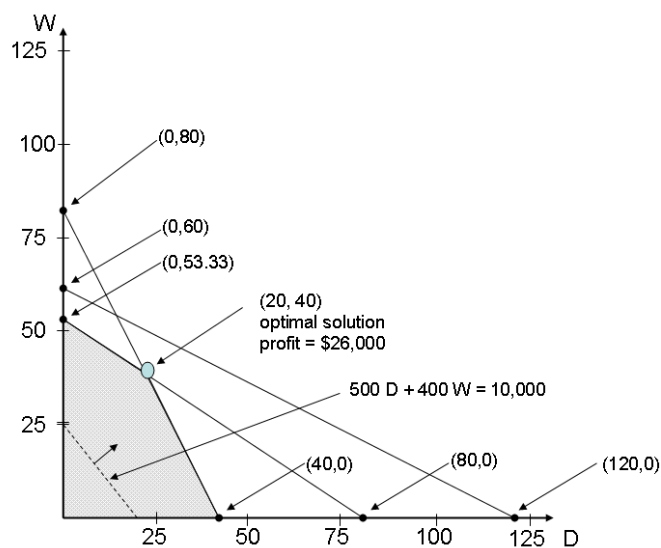
24.  $X_1$  = acres planted in watermelons,  $X_2$  = acres planted in cantaloupes

$$\begin{array}{ll} \text{MAX} & 256 X_1 + 284.5 X_2 \\ \text{ST} & 50 X_1 + 75 X_2 \leq 6000 \\ & X_1 + X_2 \leq 100 \\ & X_1, X_2 \geq 0 \end{array}$$



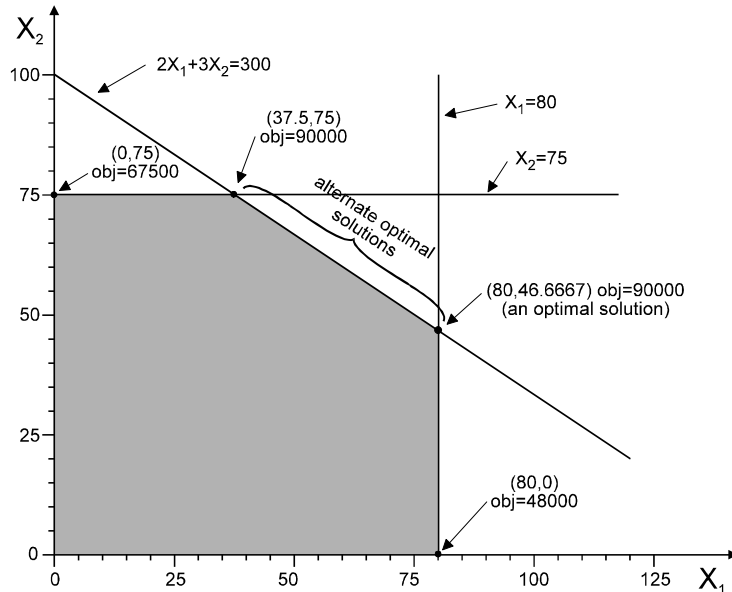
25.  $D$  = number of doors produced,  $W$  = number of windows produced

$$\begin{array}{ll} \text{MAX} & 500 D + 400 W \\ \text{ST} & 1 D + 0.5 W \leq 40 \\ & 0.5 D + 0.75 W \leq 40 \\ & 0.5 D + 1 W \leq 60 \\ & D, W \geq 0 \end{array}$$



26.  $X_1$  = number of desktop computers,  $X_2$  = number of laptop computers

$$\begin{array}{ll} \text{MAX} & 600 X_1 + 900 X_2 \\ \text{ST} & 2 X_1 + 3 X_2 \leq 300 \\ & X_1 \leq 80 \\ & X_2 \leq 75 \\ & X_1, X_2 \geq 0 \end{array}$$



### Case 2-1: For The Lines They Are A-Changin'

- 200 pumps, 1566 labor hours, 2712 feet of tubing.
- Pumps are a binding constraint and should be increased to 207, if possible. This would increase profits by \$1,400 to \$67,500.
- Labor is a binding constraint and should be increased to 1800, if possible. This would increase profits by \$3,900 to \$70,000.
- Tubing is a non-binding constraint. They've already got more than they can use and don't need any more.
- 9 to 8: profit increases by \$3,050  
8 to 7: profit increases by \$850  
7 to 6: profit increases by \$0
- 6 to 5: profit increases by \$975  
5 to 4: profit increases by \$585  
4 to 3: profit increases by \$390
- 12 to 13: profit changes by \$0  
13 to 14: profit decreases by \$760  
14 to 15: profit decreases by \$1,440

8. 16 to 17: profit changes by \$0  
17 to 18: profit changes by \$0  
18 to 19: profit decreases by \$400
9. The profit on Aqua-Spas can vary between \$300 and \$450 without changing the optimal solution.
10. The profit on Hydro-Luxes can vary between \$233.33 and \$350 without changing the optimal solution.

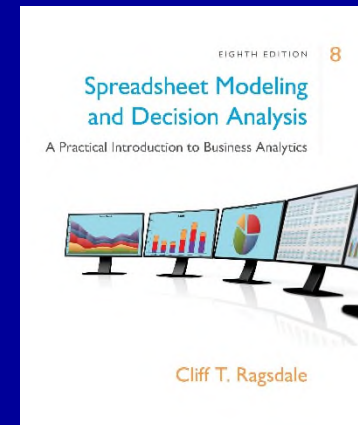
# *Spreadsheet Modeling & Decision Analysis*

A Practical Introduction to  
Business Analytics

8<sup>th</sup> edition

Cliff T. Ragsdale

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# *Chapter 2*

## Introduction to Optimization and Linear Programming

# *Introduction*

- We all face decision about how to use limited resources such as:
  - Oil in the earth
  - Land for dumps
  - Time
  - Money
  - Workers

# *Mathematical Programming...*

- MP is a field of management science that finds the optimal, or most efficient, way of using limited resources to achieve the objectives of an individual or a business.
- *a.k.a.* Optimization

# *Applications of Optimization*

- Determining Product Mix
- Manufacturing
- Routing and Logistics
- Financial Planning

# *Characteristics of Optimization Problems*

- Decisions
- Constraints
- Objectives

# *General Form of an Optimization Problem*

MAX (or MIN):  $f_0(X_1, X_2, \dots, X_n)$

Subject to:  $f_1(X_1, X_2, \dots, X_n) \leq b_1$

$\vdots$

$f_k(X_1, X_2, \dots, X_n) \geq b_k$

$\vdots$

$f_m(X_1, X_2, \dots, X_n) = b_m$

Note: If all the functions in an optimization are linear, the problem is a Linear Programming (LP) problem

# *Linear Programming (LP) Problems*

$$\text{MAX (or MIN): } c_1X_1 + c_2X_2 + \dots + c_nX_n$$

$$\text{Subject to: } a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n \leq b_1$$

$$:$$

$$a_{k1}X_1 + a_{k2}X_2 + \dots + a_{kn}X_n \geq b_k$$

$$:$$

$$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n = b_m$$

## *An Example LP Problem*

Blue Ridge Hot Tubs produces two types of hot tubs: Aqua-Spas & Hydro-Luxes.

	Aqua-Spa	Hydro-Lux
Pumps	1	1
Labor	9 hours	6 hours
Tubing	12 feet	16 feet
Unit Profit	\$350	\$300

There are 200 pumps, 1566 hours of labor, and 2880 feet of tubing available.

## *5 Steps In Formulating LP Models:*

1. Understand the problem.
2. Identify the decision variables.

$X_1$ =number of Aqua-Spas to produce

$X_2$ =number of Hydro-Luxes to produce

3. State the objective function as a linear combination of the decision variables.

$$\text{MAX: } 350X_1 + 300X_2$$

## *5 Steps In Formulating LP Models*

*(continued)*

4. State the constraints as linear combinations of the decision variables.

$$1X_1 + 1X_2 \leq 200 \quad \text{ } \} \text{ pumps}$$

$$9X_1 + 6X_2 \leq 1566 \quad \text{ } \} \text{ labor}$$

$$12X_1 + 16X_2 \leq 2880 \quad \text{ } \} \text{ tubing}$$

5. Identify any upper or lower bounds on the decision variables.

$$X_1 \geq 0$$

$$X_2 \geq 0$$

## *LP Model for Blue Ridge Hot Tubs*

$$\text{MAX: } 350X_1 + 300X_2$$

$$\text{S.T.: } 1X_1 + 1X_2 \leq 200$$

$$9X_1 + 6X_2 \leq 1566$$

$$12X_1 + 16X_2 \leq 2880$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

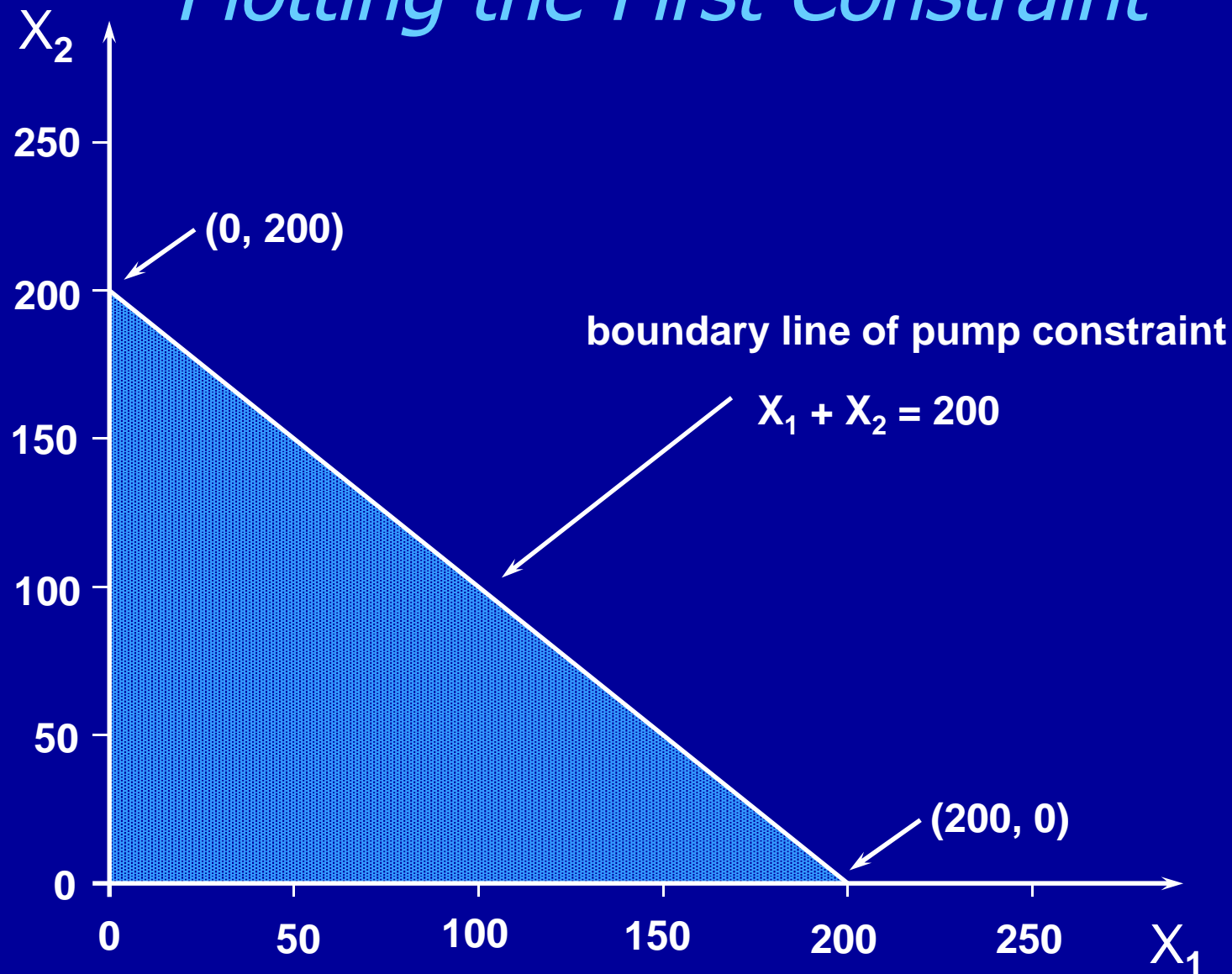
## *Solving LP Problems: An Intuitive Approach*

- Idea: Each Aqua-Spa ( $X_1$ ) generates the highest unit profit (\$350), so let's make as many of them as possible!
- How many would that be?
  - Let  $X_2 = 0$ 
    - 1st constraint:  $1X_1 \leq 200$
    - 2nd constraint:  $9X_1 \leq 1566$  or  $X_1 \leq 174$
    - 3rd constraint:  $12X_1 \leq 2880$  or  $X_1 \leq 240$
- If  $X_2=0$ , the maximum value of  $X_1$  is 174 and the total profit is  $\$350 \cdot 174 + \$300 \cdot 0 = \$60,900$
- This solution is *feasible*, but is it *optimal*?
- **No!**

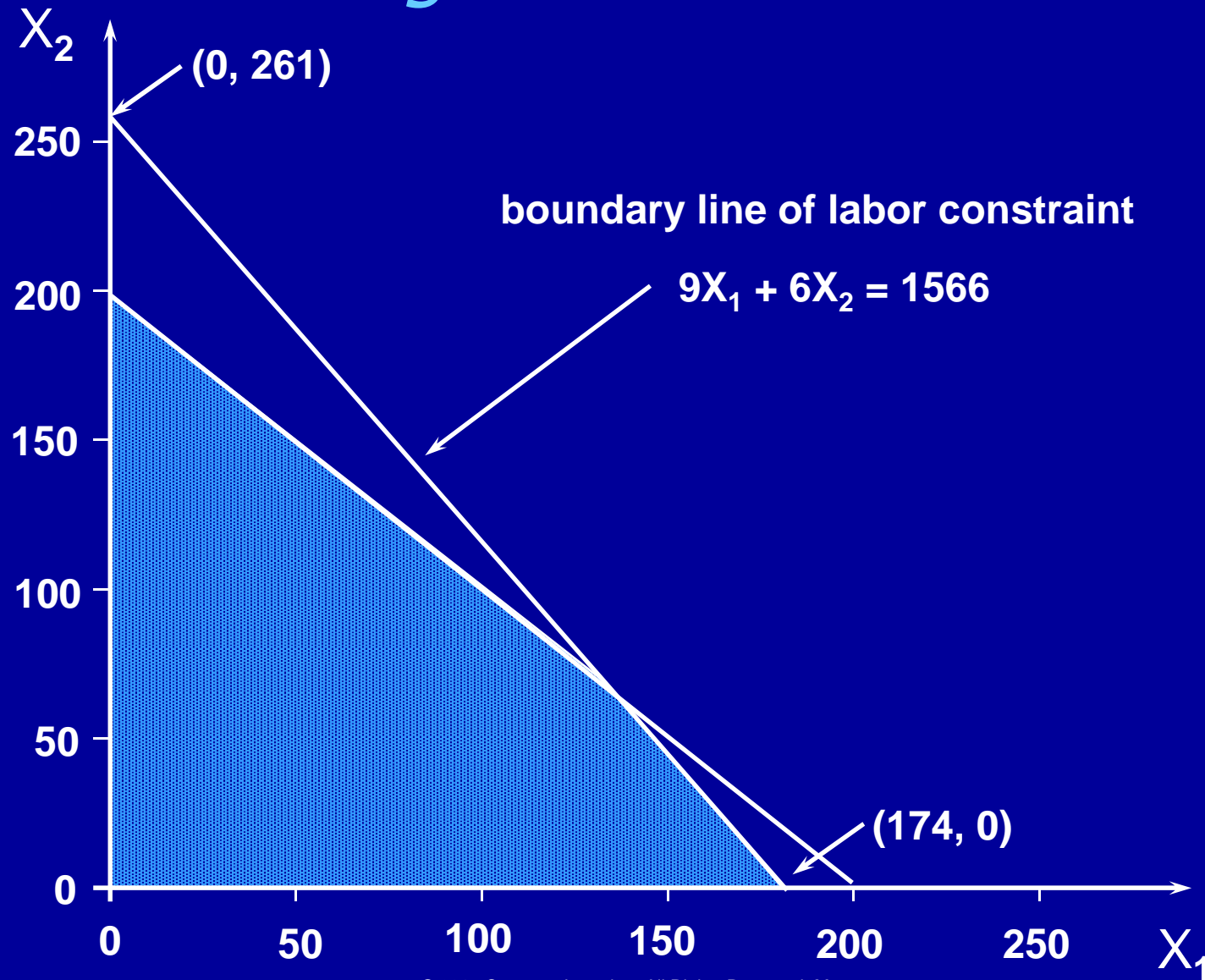
## *Solving LP Problems: A Graphical Approach*

- The constraints of an LP problem defines its feasible region.
- The best point in the feasible region is the optimal solution to the problem.
- For LP problems with 2 variables, it is easy to plot the feasible region and find the optimal solution.

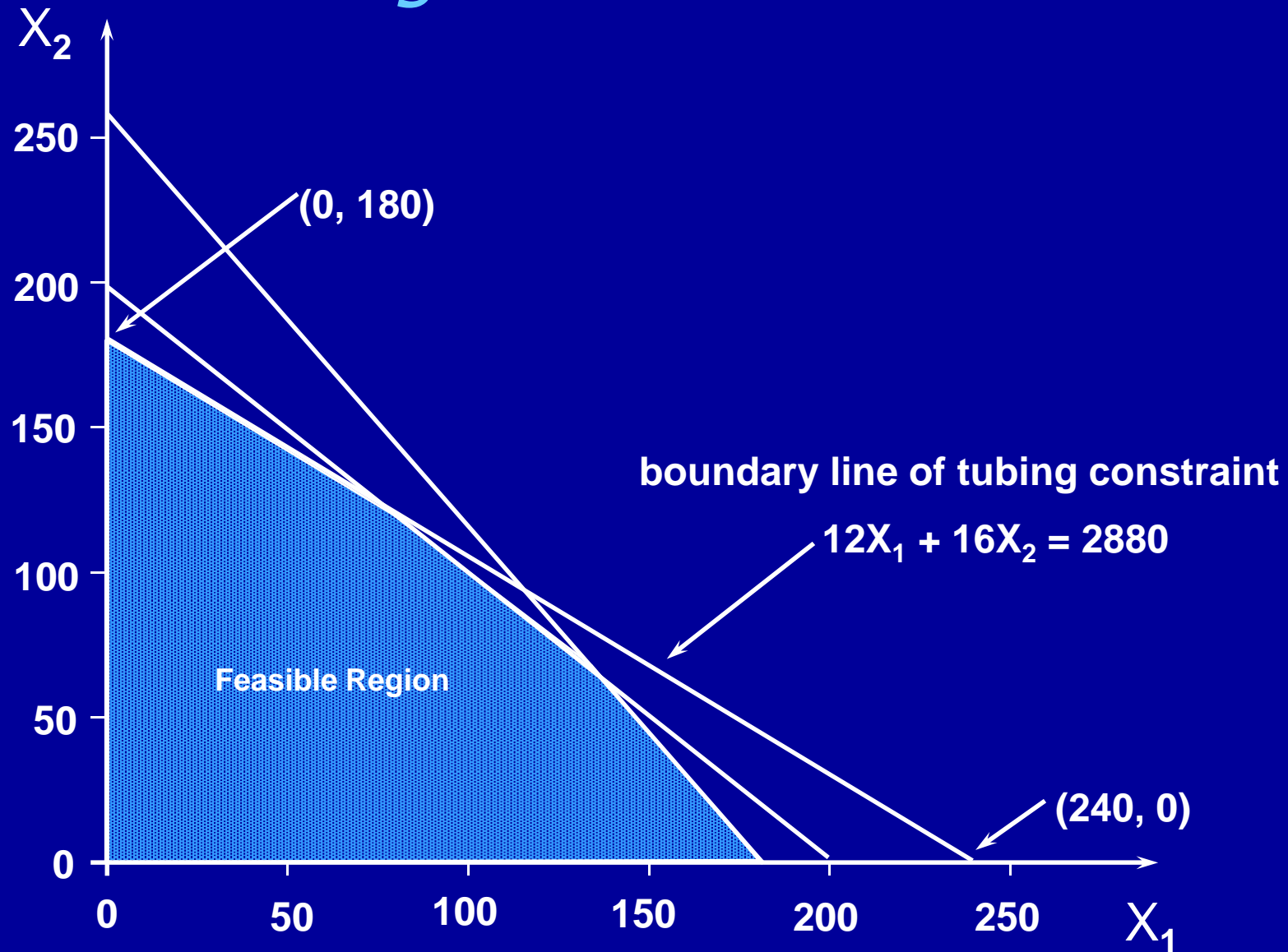
## *Plotting the First Constraint*



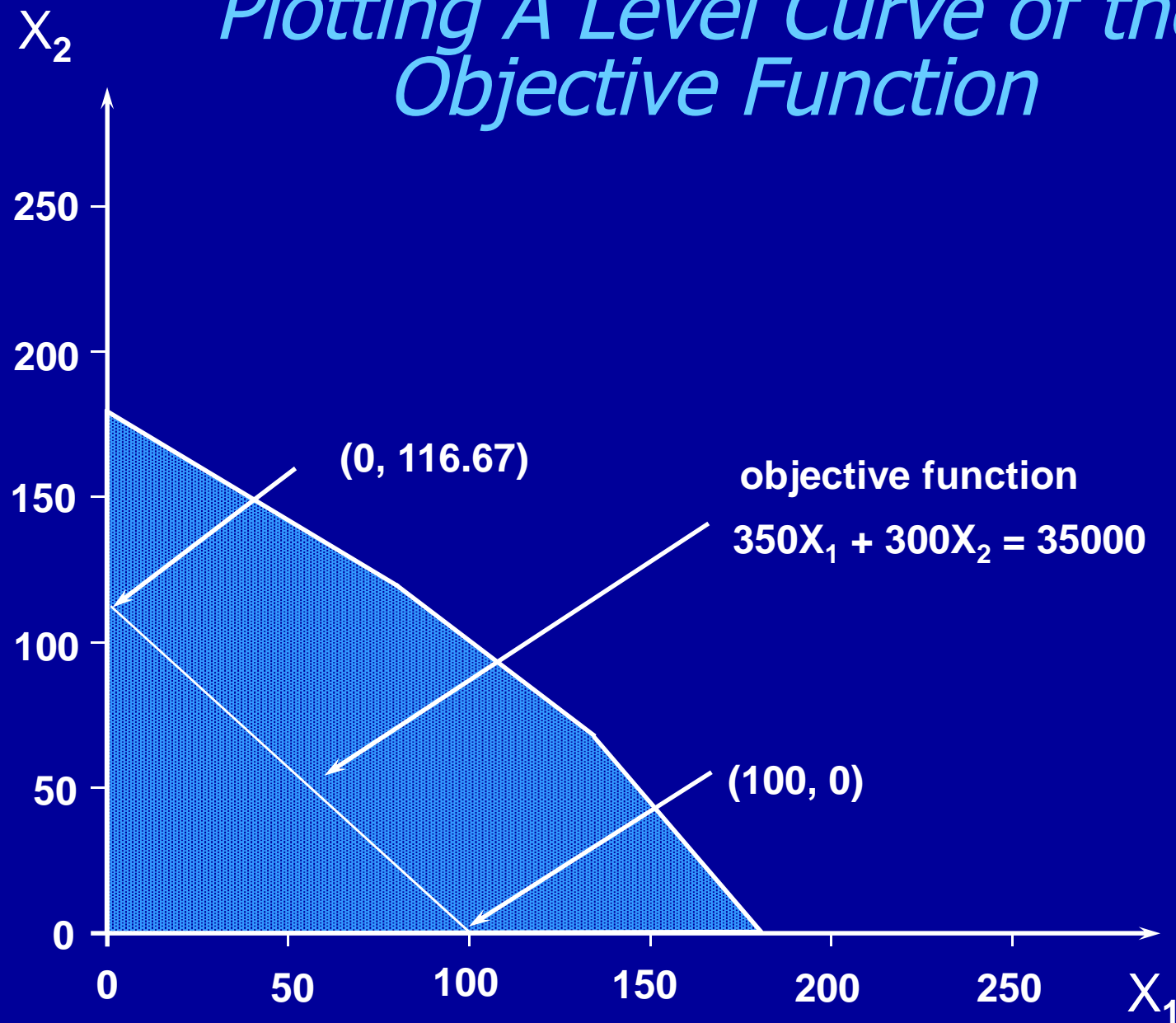
## *Plotting the Second Constraint*



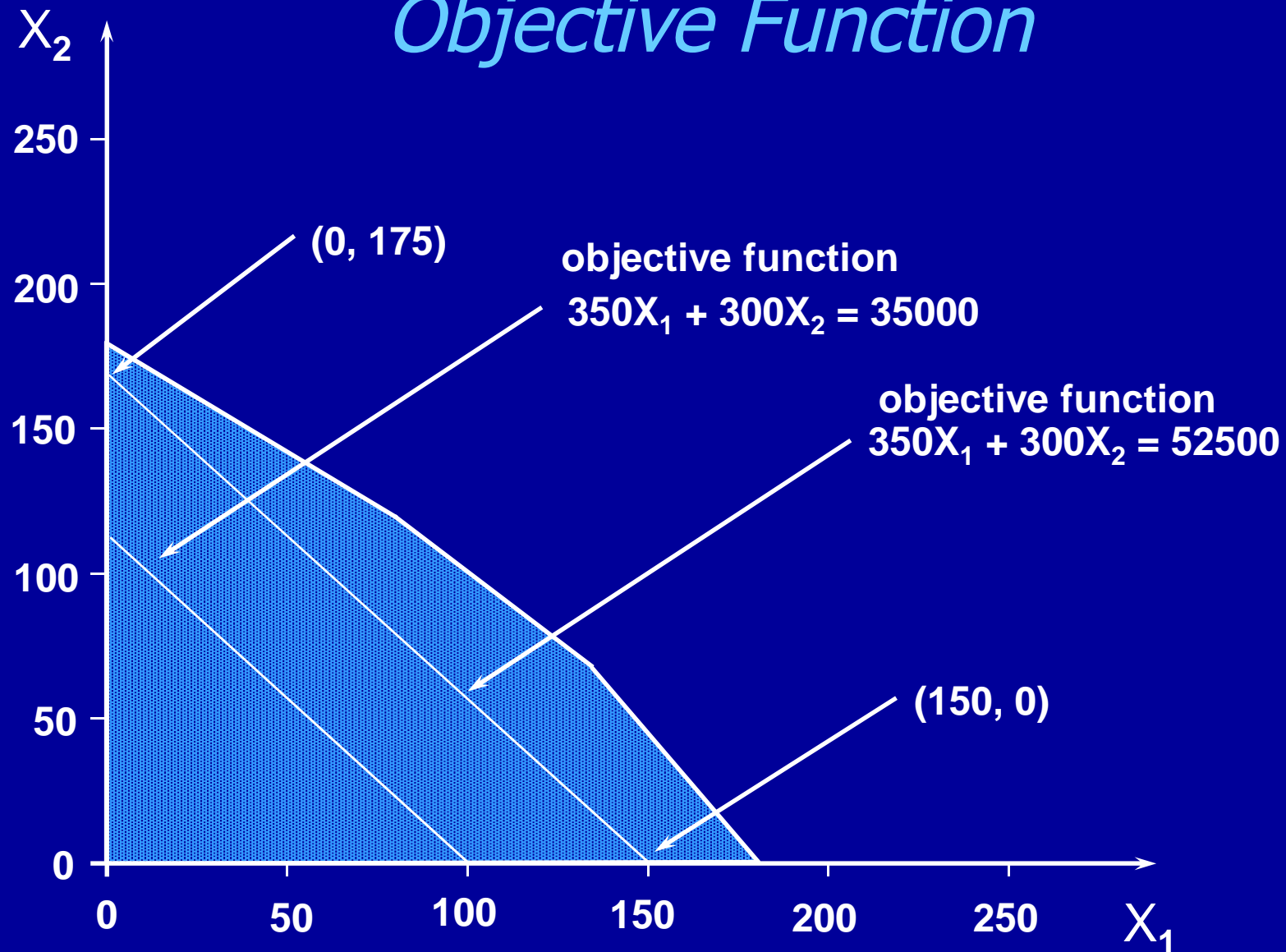
## *Plotting the Third Constraint*



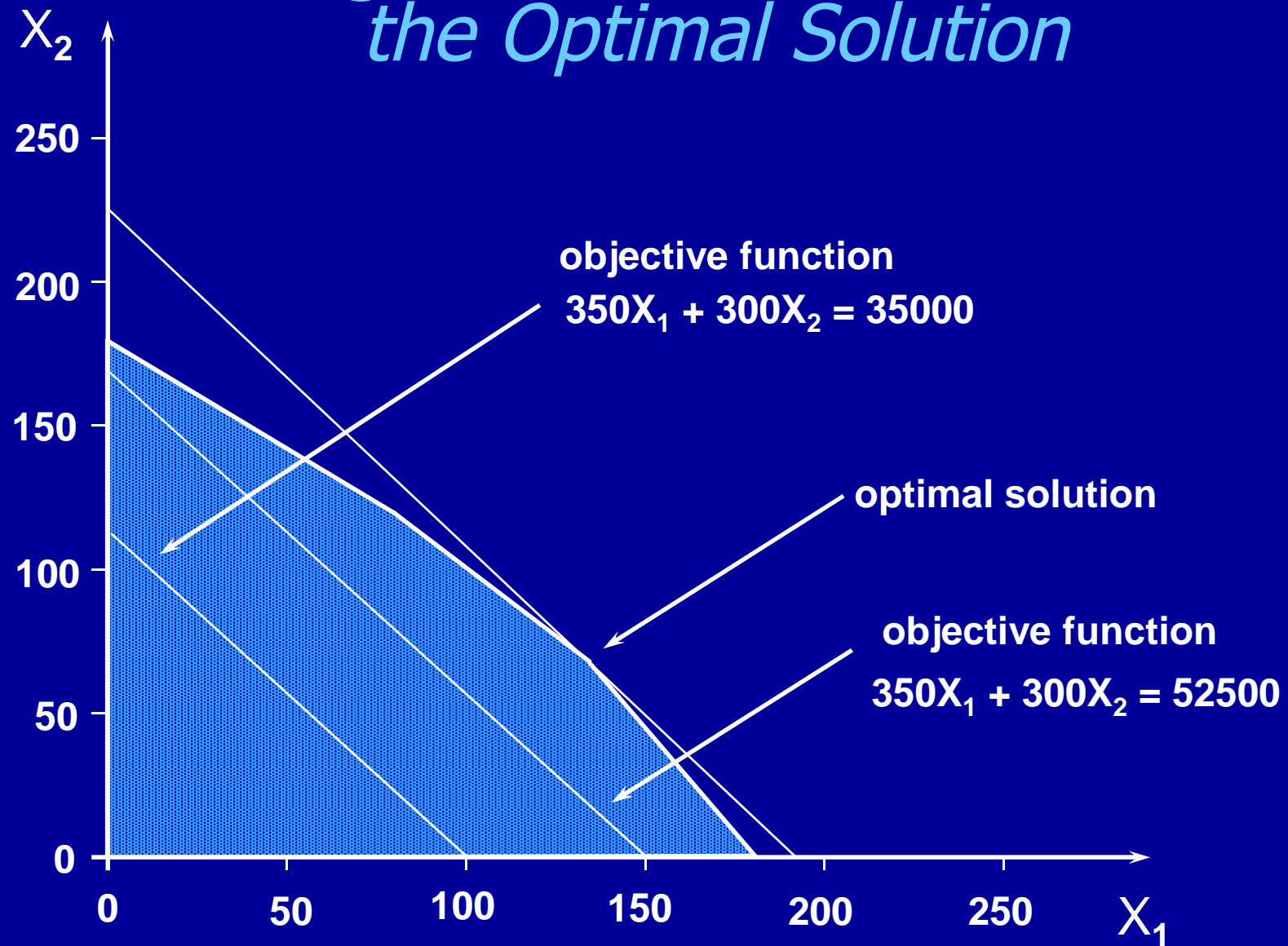
## *Plotting A Level Curve of the Objective Function*



## *A Second Level Curve of the Objective Function*



## *Using A Level Curve to Locate the Optimal Solution*



## *Calculating the Optimal Solution*

- The optimal solution occurs where the “pumps” and “labor” constraints intersect.
- This occurs where:

$$X_1 + X_2 = 200 \quad (1)$$

$$\text{and } 9X_1 + 6X_2 = 1566 \quad (2)$$

- From (1) we have,  $X_2 = 200 - X_1$  (3)

- Substituting (3) for  $X_2$  in (2) we have,

$$9X_1 + 6(200 - X_1) = 1566$$

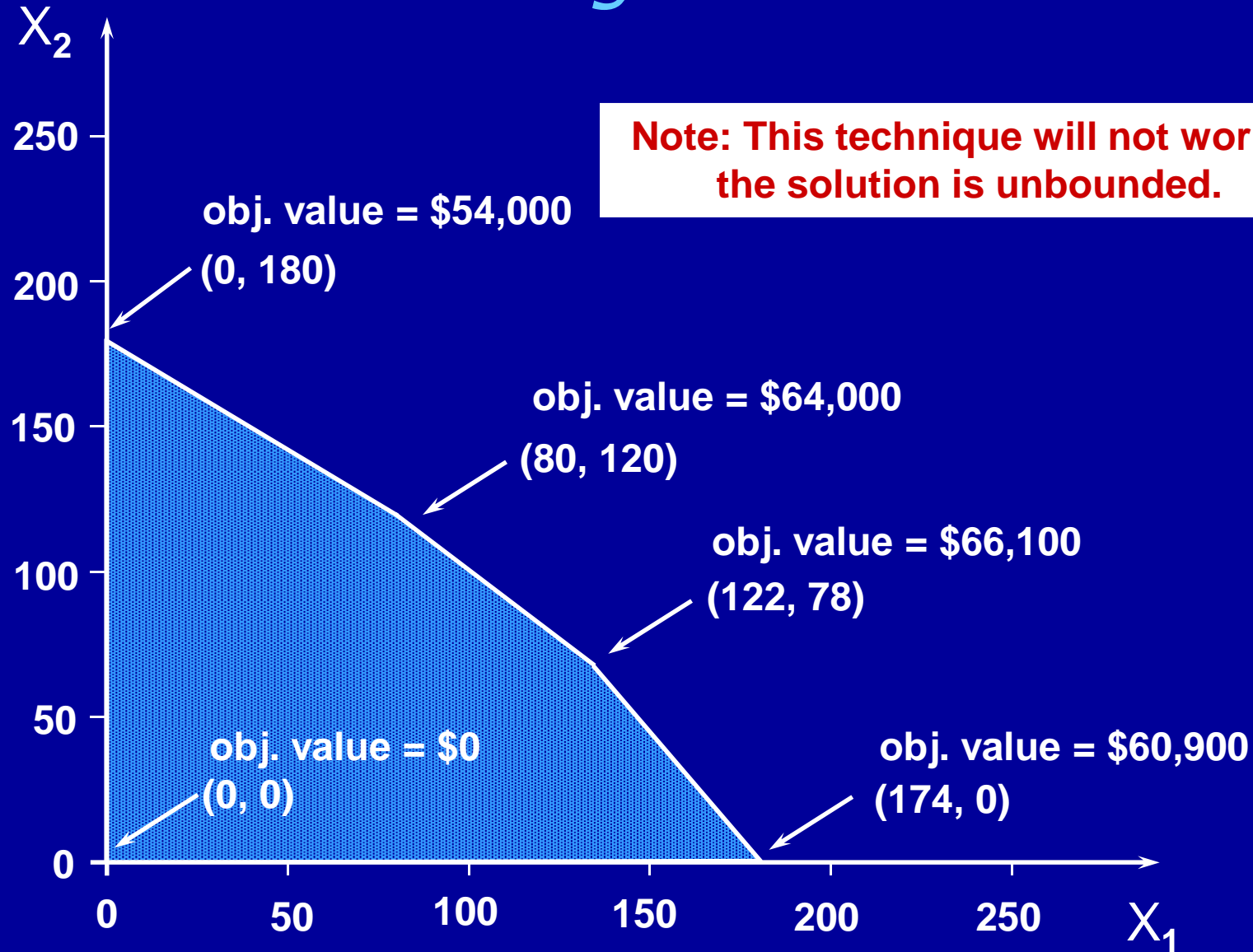
$$\text{which reduces to } X_1 = 122$$

- So the optimal solution is,

$$X_1 = 122, X_2 = 200 - X_1 = 78$$

$$\text{Total Profit} = \$350 \times 122 + \$300 \times 78 = \$66,100$$

# Enumerating The Corner Points



## *Summary of Graphical Solution to LP Problems*

1. Plot the boundary line of each constraint
2. Identify the feasible region
3. Locate the optimal solution by either:
  - a. Plotting level curves
  - b. Enumerating the extreme points

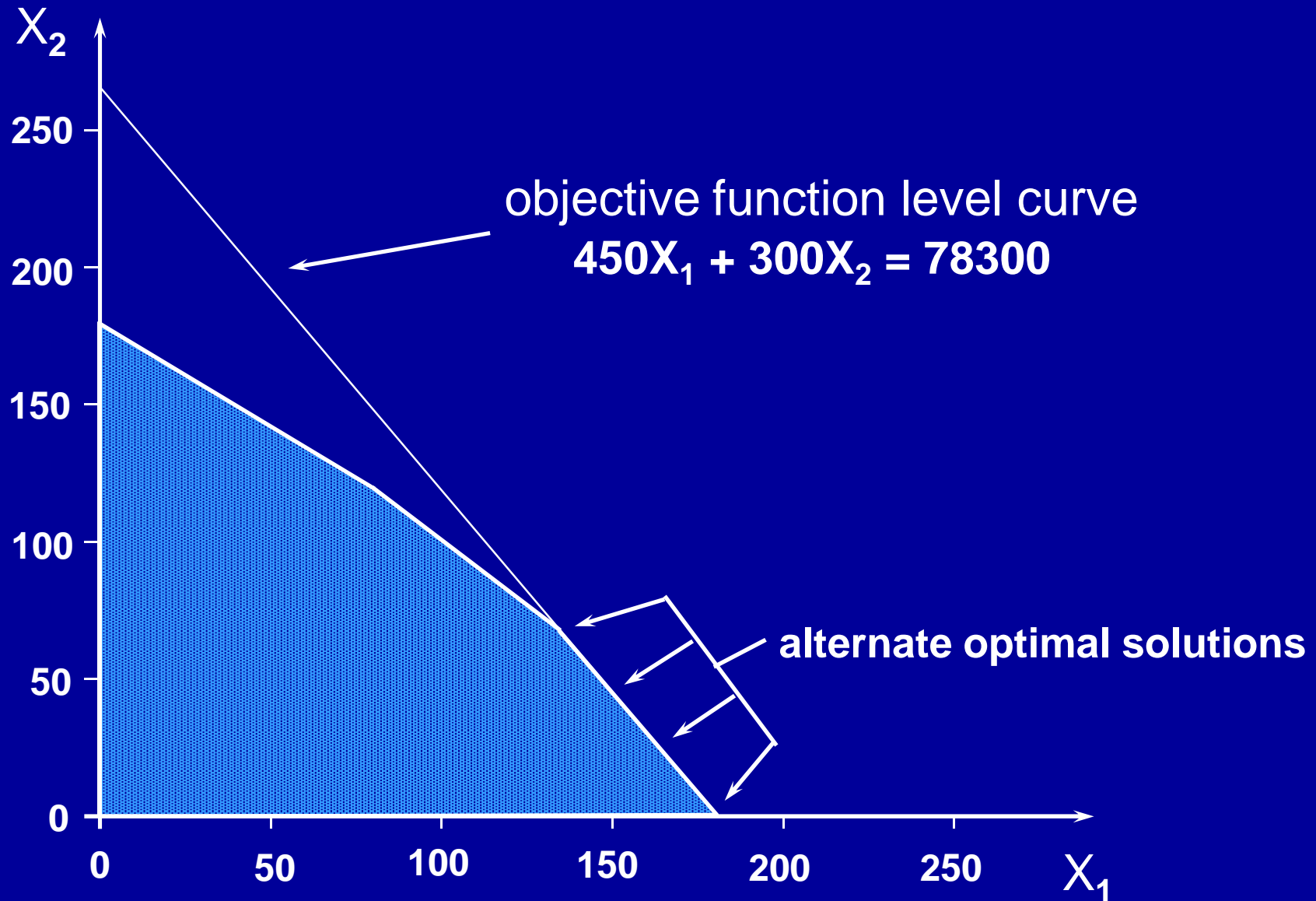
# *Understanding How Things Change*

See file [Fig2-8.xlsm](#)

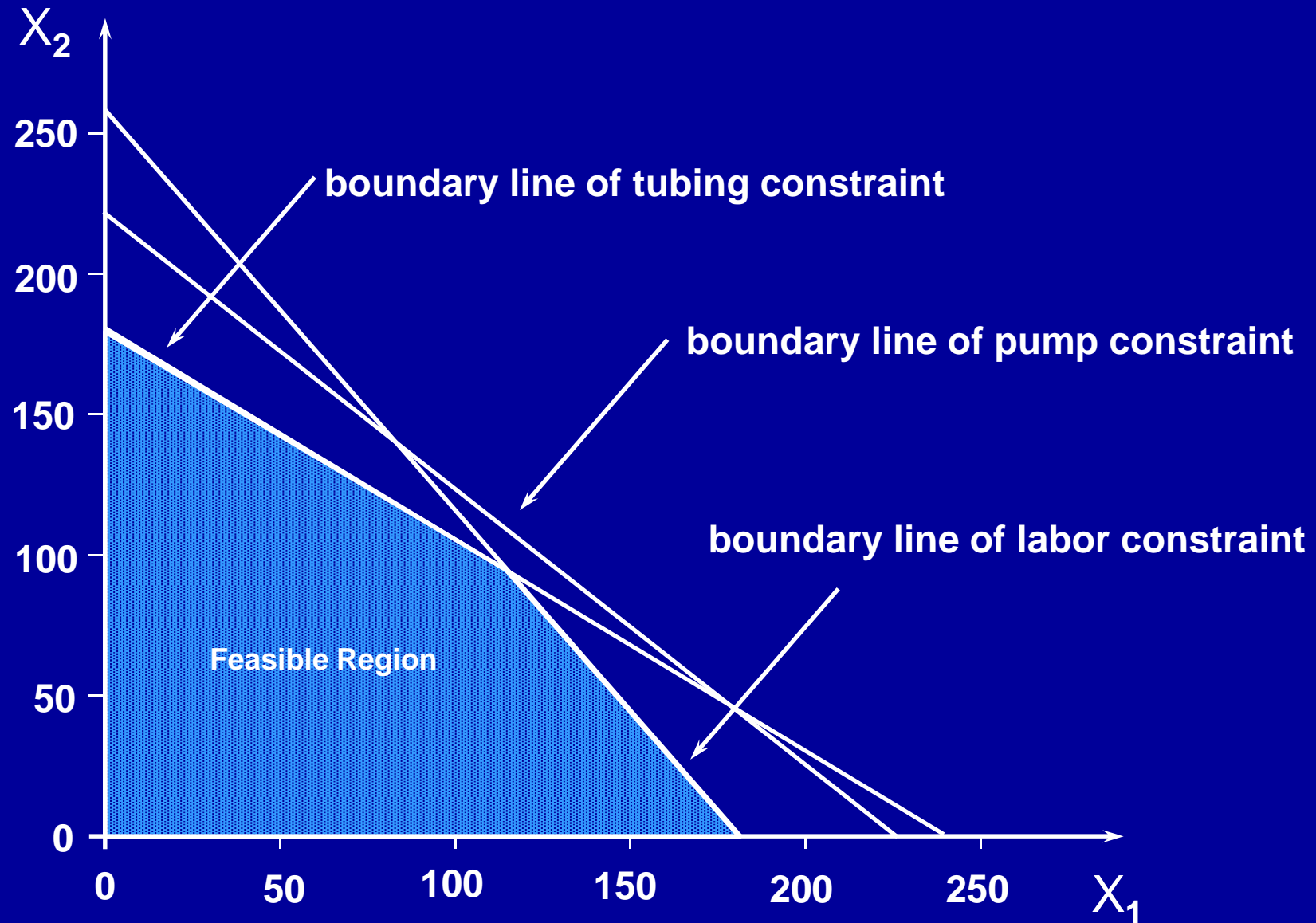
## *Special Conditions in LP Models*

- A number of anomalies can occur in LP problems:
  - Alternate Optimal Solutions
  - Redundant Constraints
  - Unbounded Solutions
  - Infeasibility

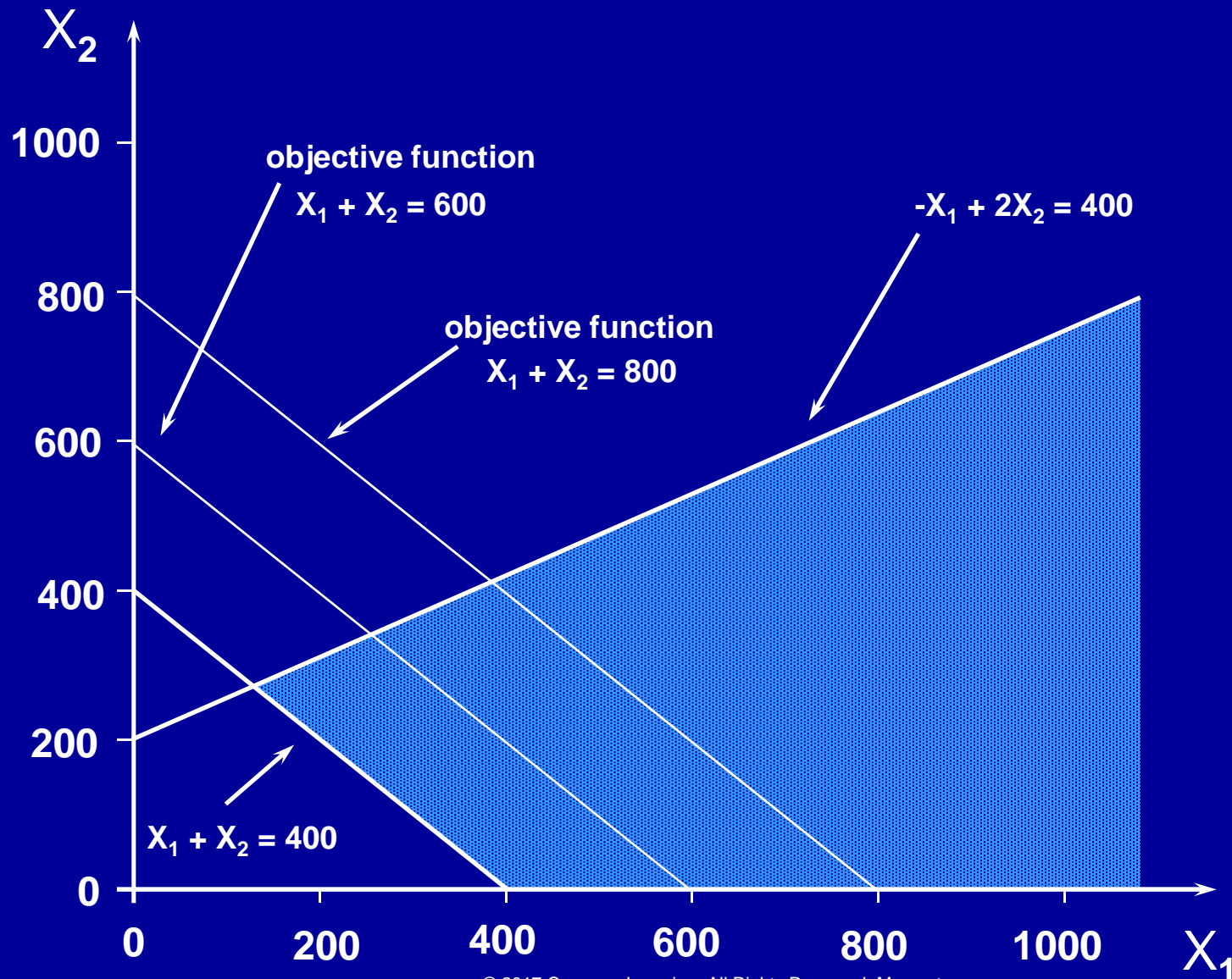
# *Example of Alternate Optimal Solutions*



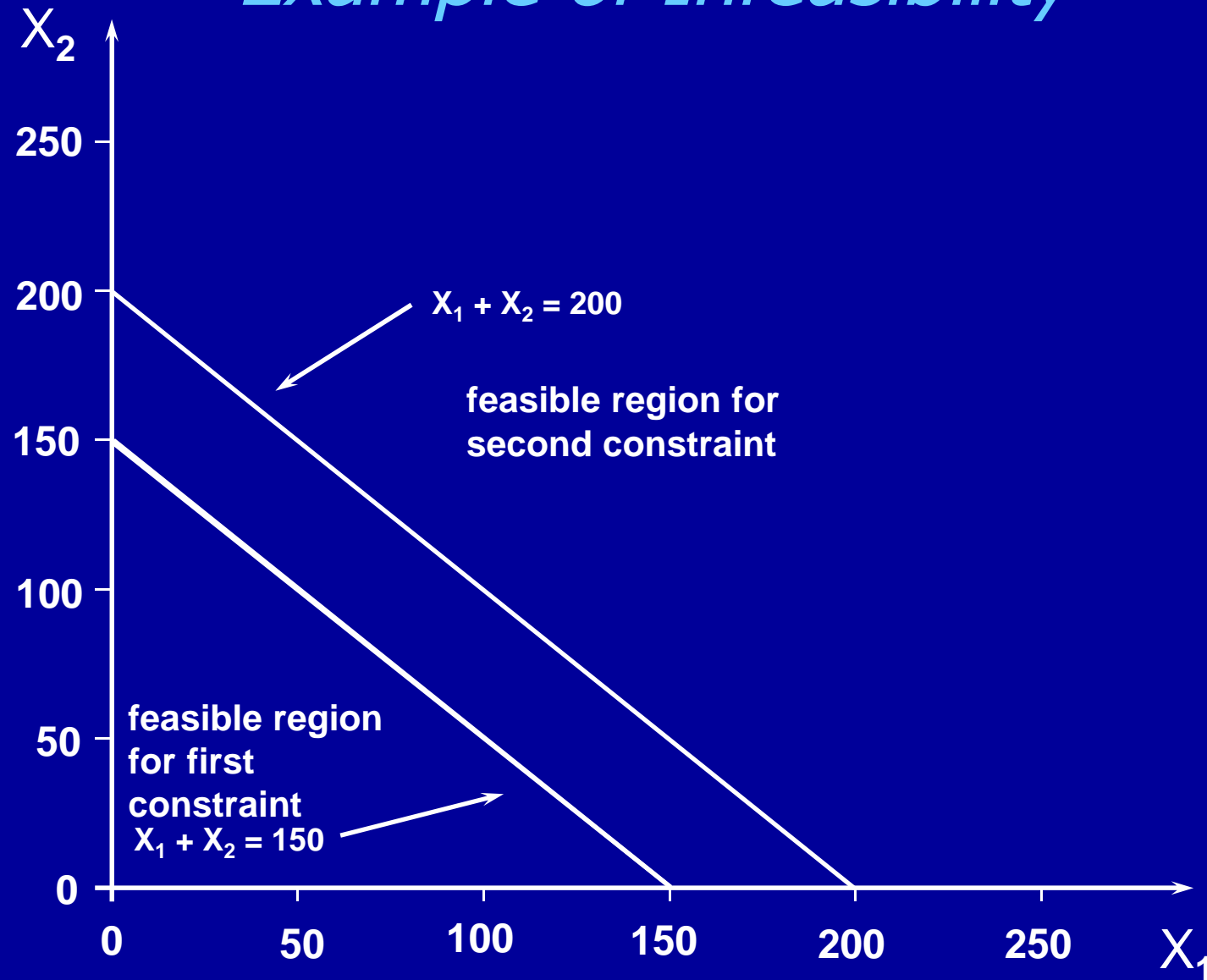
## *Example of a Redundant Constraint*



# *Example of an Unbounded Solution*



## *Example of Infeasibility*



# *End of Chapter 2*

The Analytic Solver Platform  
software featured in this book is  
provided by Frontline Systems.

<http://www.solver.com>

